ON AN OLD PROBLEM OF KNASTER

Janusz J. Charatonik

Universidad Nacional Autónoma de México (México, D. F., México)

When the definition of dendroids was started to be formulated, 1958/1959 and in the early sixties of the previous century in Wrocław Higher Topology Seminar of the Polish Academy of Sciences (conducted by Bronisław Knaster), Knaster saw this class of arcwise connected curves as ones that can be retracted onto their subdendrites or even onto their subtrees under small retractions, i.e., retractions that move points a little. Later the contemporary definition of a *dendroid* as an arcwise connected and hereditarily unicoherent continuum has been formulated and commonly accepted because it is much more convenient to work with. But the problem if the two concepts coincide is still open, and became a classic question in continuum theory (see for example [7, 10.58, p. 192]).

Question 1. Let X be a dendroid. Do there exist, for each $\varepsilon > 0$, a tree (a dendrite) $T \subset X$ and a retraction $r : X \to T$ with $d(x, r(x)) < \varepsilon$ for each point $x \in X$?

Some partial positive answers can be found in [6, Theorem 2, p. 261] for smooth dendroids, and in [5, Theorem 1, p. 120] for fans. See also [4].

Recall that if the assumption on the mapping of being a retraction onto a tree T contained in X is omitted, then the answer to the question is affirmative, since each dendroid, being a tree-like continuum, admits for each $\varepsilon > 0$ an ε-mapping onto a tree, see [3].

The property of having "small" retractions onto trees is related to the following concept of an *approximative absolute retract*. A compact metric space X is called an *approximative absolute retract* (abbr. AAR) if, whenever X is embedded into another metric space Y, then forevery $\varepsilon > 0$ there exists a mapping $f_\varepsilon : Y \to X$ such that $d(x, f_\varepsilon(x)) < \varepsilon$ for each $x \in X$. Since trees are absolute retracts, it is clear that any compact space that admits "small" retractions onto trees must be an AAR.

The two following questions are closely related to Knaster's question discussed here. They are formulated at the end of [2].

Question 2. Is every dendroid an AAR?

Question 3. Is each dendroid the inverse limit of an inverse sequence of (nested) trees with retractions as bonding mappings?

More information on dendroids and some open problems related to them is in [1].

References

1. J. J. Charatonik, *On acyclic curves. A survey of results and problems*, Bol. Soc. Mat. Mexicana (3) **1** (1995), 1–39.
2. J. J. Charatonik and J. R. Prajs, AANR *spaces and absolute retracts for tree-like continua*, preprint.
3. H. Cook, *Tree-likeness of dendroids and λ-dendroids*, Fund. Math. vol 68 (1970), 19–22.
4. C. A. Eberhart and J. B. Fugate, *Approximating continua from within*, Fund. Math. **72** (1971), 223–231.
5. J. B. Fugate, *Retracting fans onto finite fans*, Fund. Math. **71** (1971), 113–125.
6. J. B. Fugate, *Small retractions of smooth dendroids onto trees*, Fund. Math. **71** (1971), 255–262.
7. S. B. Nadler, Jr., *Continuum theory*, M. Dekker, New York, Basel and Hong Kong, 1992.

Instituto de Matemáticas, UNAM, Circuito Exterior, Ciudad Universitaria, 04510 México, D. F., México

E-mail address: jjc@math.unam.mx