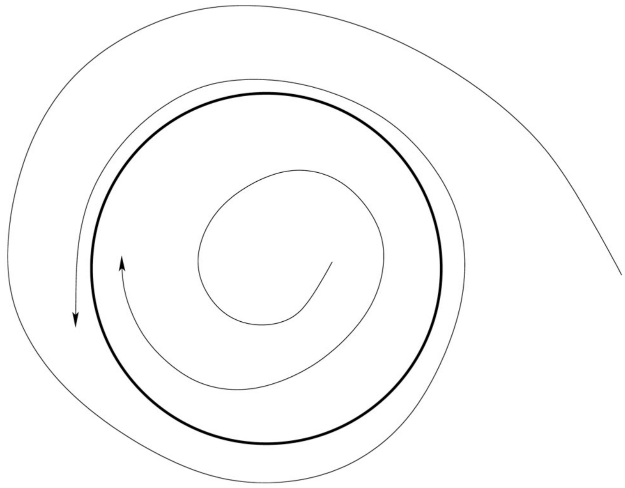
Property of Kelley in hyperspaces

A metric continuum X has *the property of Kelley* if for every sequence $\{x_n\}_{n=1}^{\infty}$ converging to a point $x \in X$, for every continuum K containing the point x, there are continua K_n containing points x_n respectively such that $\operatorname{Lim} K_n = K$. The property of Kelley is equivalent to the continuity of the function $F : X \to C(C(X))$ defined by $F(x) = \{A \in C(X) : x \in A\}$.

It is known that the property of Kelley is not preserved under Cartesian products (see [R. W. Wardle, *On a property of J. L. Kelley*, Houston J. Math., 3 (1977), 291 - 299]), nor by the hyperspace operation 2^X (see [W. J. Charatonik, *Hyperspaces and the property of Kelley*, Bull. Acad. Polon. Sci., Ser. Sci. Math. 30 (1982), 457-459]). In both cases the counterexample is the double spiral continuum as in the picture. It is important that the spirals approach the circle in different directions.



The question whether the property of Kelley for X implies one for $C(X)$ was posed by Sam B. Nadler, Jr. in his book [*Hyperspaces of sets*, Marcel Dekker, Inc, New York, NY ,1978, Question 16.37]. A partial result was obtained by H. Kato, who showed that if X has property $(\kappa)^*$, that is stronger than the property of Kelley, defined in his paper [H. Kato, *On the property of Kelley in the hyperspace and Whitney continua*, Topology Appl. 30 (1988), 165 - 174] then $C(X)$ also has property $(\kappa)^*$, and thus the property of Kelley. The double spiral continuum does not have property $(\kappa)^*$.

Another surprising result obtained by H. Kato is the following (see [H. Kato, *A note on continuous maps and the property of J. L. Kelley*, Proc. Amer. Math. Soc., 112 (1991), 1143 - 1148]): If it is true, that the property of Kelley for X implies the property of Kelley for $X \times [0, 1]$ for every continuum X, then it is also true, that the property of Kelley is preserved by the hyperspace $C(X)$ operation. Here note that we need the implication to be true for all continua, not just for a particular one. Thus the question whether the property of Kelley for X implies one for $C(X)$ leads to another important one.

Question (H. Kato 1991). Does the property of Kelley for a continuum X imply one for $X \times [0, 1]$?

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